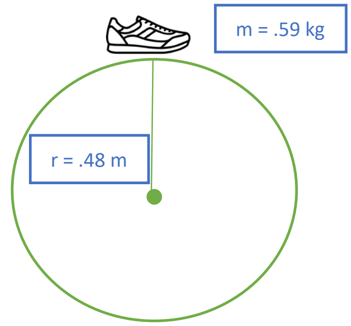
A Supreme Study of a Sneaker on a String in a Circle

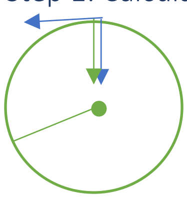
Sophie Taubman, Emily Cardwell, Judah Rosen 6th Period H. Physics

Objective Summary

In this highly advanced laboratory experiment, an old, heavy sneaker was swung by its shoelaces in a circular path at a velocity equal to that needed so at the height of its path the tension in the laces went to zero. Doing this allowed us to observe the effects of centripetal forces, for when the tension in the laces is zero the centripetal acceleration is equal to the gravitational acceleration. After finding these values, the sneaker's maximum, minimum, and tangential (average) velocities were calculated, as well as the shoe's frequency. The predicted (calculated) values were then compared to the measured ones.



Step 1: Calculating the Minimum Velocity of the Shoe



$$\mathbf{a_c = v^2 / r} \qquad \text{(Eq. 1)}$$

At top, when the tension vector = 0 (blue arrow),

the acceleration vector is equivalent because the force of tension,

which equals the force of acceleration, which always points towards the center of the curve.

So, at the top of its path, the tension vector is pointing straight "down" at the center, as is the force of gravity since gravity always points downward.

$$\mathbf{F_g = mg} \qquad \text{(Eq. 2)}$$

Since at the top of the shoe's path, the force of tension equals the force of gravity, then we can set Equation 2 equivalent to Equation 1:

$$\mathbf{a_c = v^2 / r = mg = F_g}$$

Next, the measured mass and radius values, along with the gravitational constant can be plugged into the equations:

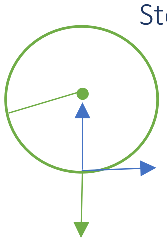
$$\mathbf{v^2/(.48\ m) = (0.59\ kg) \times (9.8\ m/s^2)}$$

$$\boxed{\mathbf{V_{top} = 1.7\ m/s}}$$

1.7 m/s is the minimum velocity of the shoe, at the top of its circular path when force of tension = 0.

Conversely, at the bottom of the circle the force of tension vector is pointing "up" at the center of the circle while the gravitational force vector is pointing in the opposite direction.

Step 2: Calculating the Maximum Velocity of the Shoe



$$PE = mgh \qquad \text{(Eq. 3)}$$

Next, using Equation 3, we can find the maximum velocity of the shoe, which occurs at the bottom:

$$PE = (0.59 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (0.48 \text{ m} \times 2)$$

$$PE = 5.6 \text{ N}$$

$$KE = \tfrac{1}{2} mv^2 \qquad \text{(Eq. 4)}$$

Next using Newton's Law of Conservation of Energy, we can set Equation 3 equal to Equation 4:

$$KE = \tfrac{1}{2} mv^2 = mgh = PE$$

$$\tfrac{1}{2} \times (0.59 \text{ kg}) \times v^2 = 5.6 \text{ N}$$

$$\boxed{v_{bottom} = 4.3 \text{ m/s}}$$

4.3 m/s is the maximum velocity of the shoe, at the bottom of its circular path.

Step 3: Calculating the Tangential (average) Velocity of the Shoe

$$v_{average} = (v_{top} + v_{bottom}) / 2 \qquad \text{(Eq. 5)}$$

By adding the maximum and minimum velocities of the shoe, we can find its tangential (average) velocity:

$$v_{average} = ((1.7 \text{ m/s}) + (4.3 \text{ m/s})) / 2$$

$$\boxed{v_{average} = 3 \text{ m/s}}$$

3 m/s is the shoe's tangential velocity.

Step 4: Calculating the Shoe's Frequency

$$F = v_{average}/2\pi r \qquad \text{(Eq. 6)}$$

Using Equation 6 and the tangential velocity found in Part 3, we can find the frequency of the shoe, meaning how many revolutions of the path the shoe goes in one second--the length of the path being the circles circumference:

$$F = (3 \text{ m/s})/2\pi(0.48 \text{ m})$$

$$\boxed{F_{predicted} = 0.99 \text{ rps}}$$

The shoe goes 0.99 revolutions per second.

Step 5: Calculating Frequency of Shoe from Measured Values

Using Apple's handy-dandy stopwatch feature on the iPhone, we counted how many revolutions the shoe did, going at the velocity needed so that the tension at the top of its path was zero, in one minute.

$$\textbf{F = revolutions/1 min x 1 min/60 seconds} \qquad \text{(Eq. 7)}$$

Applying the conversion factor given in equation 7, we found that the shoe's frequency was:

F = 64 rev/1 min x 1 min/60 seconds

$$\boxed{\mathbf{F_{measured} = 1.1\ rps}}$$

1.1 revolutions per second was the measured frequency of the shoe.

Step 6: Comparing Predicted Values with Measured Values—Calculating Percent Error

$$\textbf{\% Error} = ((F_{measured} - F_{predicted}) / F_{predicted}) \times 100 \qquad \text{(Eq. 8)}$$

Using Equation 8, the Percent Error, and therefore accuracy, of the predicted value can be found:

% Error = ((1.1 rps – 0.99 rps)/(0.99 rps)) x 100

$$\boxed{\textbf{\% Error = 11 \%}}$$

The difference between the measured frequency and predicted frequency was 11 percent, meaning the actual frequency was + or - .11 (11 %) of the predicted frequency of 0.99 rps.

Step 7: Reflect!

After doing this lab, not only was a greater understanding of centripetal force gained, but also a new purpose for an old, untouched, heavy shoe was successfully found!